Seismic Signal Source Separation

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Abstract — This paper gives a brief description of the DUET blind source separation algorithm, and applies the technique to synthetic seismic signals, to separate explosive sources in a volcano. The source separation method worked successfully to separate two contemporary explosive sources in a 3D structural model of Mt Etna. The sources were separated almost perfectly (with a 15dB signal to interference ratio).

Keywords — Source Separation, Geophysical Signal Processing.

I Introduction

There are two distinct types of seismic sources found in geophysical science:

- Rapid stress drop earthquakes caused by brittle material failure. These events usually occur in isolation and are identified by clear P-wave onsets.

- Continuous tremor events, often with emergent onsets. These tremor events can be found in two distinct environments:- on volcanoes and in subduction zones.

The origins of these events are still poorly understood (and may not be unique), but multiple sources are often known to be simultaneously active. This issue has not previously been addressed by conventional geophysical analysis.

This paper treats the issue of separating multiple sources as a blind source separation problem. The method used to separate the sources in this paper is the Degenerate Unmixing Estimation Technique (DUET) [1], which uses a pair of stations, separated by less than half a wavelength of the signal.

II The DUET method

DUET was designed for blind source separation of speech signals, by taking advantage of the sparsity of speech in the time-frequency representation of sources [2, 3, 4, 5, 6]. The technique works well when there is little or no overlap between the two sources in the time-frequency domain. Seismic sources inside volcanoes can also exhibit sparsity in the time-frequency domain, and this property allows DUET to successfully separate seismic signals. The assumptions made and the algorithm are described in the following section.

a) Assumptions

a).1 Anechoic mixing

Consider a mixture of \( N \) source signals, \( s_j(t), j = 1, ..., N \), is received at a pair of stations where only the direct path is present. In this case, without loss of generality, we can absorb the attenuation and delay parameters of the first mixture, \( x_1(t) \), into the definition of the sources. As such, the two
an echoic mixtures can be expressed as:

\[ x_1(t) = \sum_{j=1}^{N} s_j(t), \]  
\[ x_2(t) = \sum_{j=1}^{N} a_j s_j(t - \delta_j), \]

where \( N \) is the number of sources, \( \delta_j \) is the arrival delay between the stations, and \( \delta_j \) is a relative attenuation factor corresponding to the ratio of the attenuations of the paths between sources and stations. The echoic mixing model is not realistic, in that it does not represent echoes, (that is, multiple paths from each source to each mixture). In spite of this limitation, however, the DUET method based on this model has proven to be quite robust, even when applied to echoic mixtures.

a).2 W-Disjoint Orthogonality

We call two functions \( s_j(t) \) and \( s_k(t) \) W-disjoint orthogonal if, for a given windowing function \( W(t) \), the supports of the windowed Fourier transforms of \( s_j(t) \) and \( s_k(t) \) are disjoint. The windowed Fourier transform of \( s_j(t) \) is defined as:

\[ \hat{s}_j(\tau, \omega) := \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} W(t - \tau)s_j(t)e^{-i\omega t}dt. \]  

(3)

So the W-disjoint orthogonality assumption can be stated as:

\[ \hat{s}_j(\tau, \omega)\hat{s}_k(\tau, \omega) = 0, \forall \tau, \omega, \forall j \neq k. \]  

(4)

This assumption is the mathematical idealization of the condition, that it is likely that every time-frequency point in the mixture, with significant energy, is dominated by the contribution of one source. Note that, if \( W(t) \equiv 1 \), \( \hat{s}_j(\tau, \omega) \) becomes the Fourier transform of \( s_j(t) \), which we will denote \( \hat{s}_j(\omega) \). In this case W-disjoint orthogonality can be expressed as:

\[ \hat{s}_j(\omega)\hat{s}_k(\omega) = 0, \forall j \neq k, \forall \omega, \]  

(5)

which we call disjoint orthogonality.

W-disjoint orthogonality is crucial to DUET, because it allows for the separation of a mixture into its component sources using a binary mask. Consider the mask which is the indicator function for the support of \( \hat{s}_j \):

\[ M_j(\tau, \omega) = \begin{cases} 1 & \hat{s}_j(\tau, \omega) \neq 0 \\ 0 & \text{otherwise.} \end{cases} \]  

(6)

\( M_j \) separates \( \hat{s}_j \) from the mixture via

\[ \hat{s}_j(\tau, \omega) = M_j(\tau, \omega)\hat{x}_1(\tau, \omega), \forall \tau, \omega \]  

(7)

where \( \hat{x}_1(\tau, \omega) \) is the time-frequency representation of \( x_1 \). As such, if we could determine the masks which are the indicator functions for each source, we can separate the sources by partitioning. The question is, how do we determine the masks? As we will shortly see, the answer involves labeling each time-frequency point with delay differences that explain the time-frequency phase between the two mixtures. These delays cluster into groups, one group for each source.

b) Algorithm

The assumptions of echoic mixing allows us to rewrite the mixing Equations (1) and (2) in the time-frequency domain as,

\[
\begin{bmatrix}
\hat{x}_1(\tau, \omega) \\
\hat{x}_2(\tau, \omega)
\end{bmatrix} = 
\begin{bmatrix}
a_1e^{-i\omega\delta_1} & \cdots & a_Ne^{-i\omega\delta_N}
\end{bmatrix}
\begin{bmatrix}
\hat{s}_1(\tau, \omega) \\
\vdots \\
\hat{s}_N(\tau, \omega)
\end{bmatrix}
\]  

(8)

If we include the further assumption of W-disjoint orthogonality, at most one source is active at every \( (\tau, \omega) \), and the mixing process can be described for each \( (\tau, \omega) \) as:

\[
\begin{bmatrix}
\hat{x}_1(\tau, \omega) \\
\hat{x}_2(\tau, \omega)
\end{bmatrix} = 
\begin{bmatrix}
a_je^{-i\omega\delta_j}
\end{bmatrix}
\begin{bmatrix}
\hat{s}_j(\tau, \omega)
\end{bmatrix}
\]  

(9)

for a given \( j \). In the above equation, \( j \) depends on \( (\tau, \omega) \), in that \( j \) is the index of the source active at \( (\tau, \omega) \). The main observation that DUET utilizes, is that the ratio of the time-frequency representations of the mixtures does not depend on the source components, but only on the mixing parameters associated with the active source component.

\[ \forall (\tau, \omega) \in \Omega_j, \frac{\hat{x}_2(\tau, \omega)}{\hat{x}_1(\tau, \omega)} = a_je^{-i\omega\delta_j}. \]  

(10)

Here, \( \Omega := \{ (\tau, \omega) : \hat{s}_j(\tau, \omega) \neq 0 \} \). The mixing parameters (the relative delay estimate in this case) associated with each time-frequency point can be calculated:

\[ \delta(\tau, \omega) = -\frac{1}{\omega} \frac{\hat{x}_2(\tau, \omega)}{\hat{x}_1(\tau, \omega)}. \]  

(11)

The steps taken to separate the sources using DUET are:

1. Construct the discrete Short-Time Fourier Transform of the signal at each station. The flowchart showing the algorithm to find the Short-Time Fourier Transform of the signal is shown in Figure 1. TFindex in the figure represents a \( (\tau, \omega) \) pairing. \( W \) is a hamming window of length winlen.

2. Take the ratio of the mixtures to extract the local delay estimate. A delay estimate will result for each time-frequency point.

3. Generate the histogram of these delays, and find the peak delay for each source. The peak delays are found using a weighted k-means clustering variant of the k-means clustering technique [7], for peak tracking.

4. For each peak found, i.e. for each source, a binary time-frequency mask is then created, based on how close the delay at each time-frequency point is to the peak delays found.

5. This mask can then be applied to each of the mixtures recorded at each station to get the time-frequency representation of the separated source recorded at each station.
6. The separated source signals are then transformed, using the inverse Short-Time Fourier Transform, back to the time domain.

The flowchart of the separation algorithm is shown in Figure 2. In the figure, the loop incrementing TF_index is the loop going through all the \((\tau, \omega)\) pairs. In this section, the assumptions made and the algorithm used to separate the sources was described. Section III describes the data that the algorithm was applied to, and shows the results of DUET on this data.

III Results

The 3D structural model for Mt Etna used in this instance, was inelastic, and its physical properties varied spatially. The lower regions of the model were constructed from the tomographic model of [8]. As tomography models are known to be poorly constrained near the Earth’s surface, a low velocity near surface gradient ranging from 1600m/s at the surface to 3000m/s at 500m depth was added to the model. The top surface of the model was defined using the actual Digital Elevation Model of Mt Etna (the topography model of Mt Etna is shown in Figure 3). The Elastic Lattice Scheme [9] was used to propagate full wavefield elastic waves

Fig. 1: Flowchart of the time-frequency analysis function.

Fig. 2: Flowchart of the DUET separation algorithm.
The topography model used for Mt Etna

(a) The topography model used for Mt Etna zoomed in at the summit

(b) The topography model used for Mt Etna

Fig. 3: The topography of Mt Etna and the summit with the station positions.

through the model. The seismic sources were explosive, and were located at depths of 280m and 1640m below the summit. One of the sources had a single frequency present, at 1Hz. The second source had a gliding frequency spectrum over time, and overlapped with the first source in some samples. The explosive source generated only P-waves (the elastic wave analogy to acoustic waves, although the shear modulus did play a role in controlling its velocity), however S-waves were generated in the model through P-to-S conversions inside the medium, and on the free surface, respectively. Dispersive surface waves were also present in the recorded signals, due to the interaction of P- and S- waves at the volcano surface. White gaussian noise was added to the signal at each station (10% of the r.m.s. of the signal). Figure 4 shows the mixtures at a pair of stations, and their time-frequency representations. The histogram of delays calculated is shown in Figure 5. Two peaks were found to be present, at -4.6205 and 0.0369, using a weighted k-means clustering variant of the k-means clustering technique. A binary time-frequency mask was then created for each source to separate them in the time-frequency domain. The inverse Short-Time Fourier Transform was then applied to the time-frequency domain of the separated signals in order to synthesize the separated sources back to the time domain. The separated sources are shown in Figure 6. In the time domain; the blue plot is the source separated by the DUET algorithm, and the red plot is the original individual source at that station. The metric used to determine the success of the algorithm mathematically was the signal to interference ratio,

$$SINR_1 = \frac{E_{1s1}}{E_{1s2}},$$

where $E_{1s1}$ is the energy in source 1 contributing to the energy in separated source 1, and $E_{1s2}$ is the energy in source 2 contributing to the energy in separated source 1. The sources were separated almost perfectly with a signal to interference ratio of 15dB. There was an overlap in the frequencies of the two sources, between samples 1000 and 1500, which results in a slight increase in the interference

(a) Mixtures from the two stations in the time domain

(b) Mixtures from the two stations in the Short Time Fourier Transform domain. A window length of 1024 samples was used, with an overlap of 512 samples, and the signal length was 2000 samples, so there are 4 windows of frequencies present.

Fig. 4: Mixtures at the two stations.
IV Conclusions and future work

In this paper, the DUET algorithm was described. This algorithm was then applied to synthetic seismic data in a 3D structural model of Mt Etna. The sources were separated almost perfectly with a signal to interference ratio of 15dB.

We plan to investigate a range of more complex sources, for example tensile crack opening, as often encountered on volcanoes, and increase the number of sources present. We also plan to apply existing array localization methods [10, 11], to the separated source signals found in an array of stations, to localize the separated sources. In addition, we will investigate the possibility of extracting small volcano earthquake events from noisy field recordings on Mt Etna using this methodology.

REFERENCES