Performance of QPSK-OFDM with LDPC and Concatenated Reed Solomon/Convolutional Coding in Outdoor Environments

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***Abstract*—This paper presents a performance comparison of two different channel coding techniques for use with an orthogonal frequency-division multiplexing (OFDM) system. The system operates in an outdoor environment and achieves coding gain by the use of a low-density parity-check (LDPC) code or a concatenated Reed Solomon - convolutional code (RS-CC). A concatenated code is a popular class of block code which operates by consecutively combining an outer code and inner code; this is a solution to the problem of finding a code with an exponential decrease in error probability as the block length increases, together with polynomial-time decoding complexity. LDPC codes are a relatively new channel code class capable of near-Shannon-limit performance over many practical communication channels. In this paper, we provide link level performance results for both LDPC and RS-CC coding used in a non-line-of-sight QPSK-OFDM system over Rayleigh fading channels. Performance results are documented for the COST 207 (Typical Urban and Bad Urban) channel as well as the Winner 2.8 scenario NLOS channel. Simulation results demonstrate that the performance of the LDPC coded system is better than the concatenated RS-CC system.**

***Keywords* – LDPC codes, concatenated Reed Solomon - convolutional codes, OFDM, COST 207 and Winner 2.8 channels.**

I INTRODUCTION

The principle of OFDM has been around for several decades, though it became a popular technology for commercial digital communication systems only in the last two decades [1]. One of the main advantages of using OFDM is that it facilitates the use of a reduced complexity equalizer. Use of error-correction coding (ECC) and interleaving helps to reduce errors resulting from spectral nulls, commonly found in multipath channels. Furthermore, ECC provides a greater immunity to the effect of multipath and signal clipping due to high signal peak to average power ratio.

As may be seen from [2], a cellular mobile system based on OFDM, which uses pilot based correction would provide a significant improvement in Rayleigh fading environment. The delay spread could cause an intersymbol interference (ISI) when adjacent data symbols overlap and interfere with each other due to different delays on different propagation paths. The maximum Doppler shift in multicarrier modulated system might cause a significant intercarrier interference (ICI) as the subchannel spacing is quite small, and this ICI may lead to a significant degradation in system performance [3]. The use of channel coding with OFDM will improve the performance significantly. This paper investigates the performance of two different coding systems with OFDM. We demonstrate the bit error rate performance of the coded LDPC and concatenated Reed Solomon–convolutional code (RS-CC) QPSK-OFDM system in non-line-of-sight (NLOS) multipath fading channels.

Low-density parity check codes a class of block code with very sparse parity check matrices. These codes perform well with an iterative probabilistic decoding algorithm. They provide near-capacity performance on a large set of data transmission and storage channels [4]. Gallager in 1960 had introduced LDPC in his doctoral thesis together with a low-complexity decoding algorithm called the sum-product algorithm, also known as *belief propagation* or *message passing* [5], [6]; the algorithm is so called because it propagates soft information, or bit value probabilities, between bit nodes and check nodes in the code’s Tanner graph.

In most coded communication systems, one can improve the error correction performance by increasing the block length of the code; however this approach is not always practical as the decoding complexity in general rises rapidly with the block length. Concatenated codes were originally proposed by Forney [7] and the idea is to cascade two or more codes in a serial manner; the overall decoding complexity is then quite manageable considering the effective block length of the concatenation. In this paper, the concatenated coding scheme employs a convolutional code as the inner code with a Viterbi decoder and a Reed Solomon code as the outer code. The LDPC code tested was a code derived from a finite Euclidean geometry. We demonstrate the performance of QPSK-OFDM over COST 207 Typical Urban (TU), COST 207 Bad Urban (BU) and Winner vehicular 2.8 scenarios with LDPC coding as well as concatenated RS/CC.

Section II outlines the OFDM system model. Section III explains various assumed parameters for this work. Bit error rate performances of the various coded QPSK-OFDM systems are presented in Section IV. Finally, conclusions are presented in Section V.

II SYSTEM MODEL

A. *OFDM system Model*

The OFDM-based communication system is a popular modern communication system (block diagrams and overview may be found in e.g. [1, 9]). The IDFT and DFT are used respectively for modulating and demodulating the data constellations replacing the banks of I/Q-modulators which would otherwise be required [9]. Input of the IDFT, N data constellation points $\{x_{i,\,k}\}$ are presented, where N is the number of DFT points. (i is an index on the subcarrier; k is an index of the OFDM symbol). To eliminate ICI (Inter Carrier Interference), OFDM symbol is cyclically extended by the guard interval and this ensures delayed replicas of the OFDM symbol always have an integer number of cycles within FFT intervals. As long as the delay is smaller than the guard interval, it leads to remedy from ICI as multipath signal with delays smaller than the guard interval [10]. Orthogonality may be lost when the delay spread is larger than length of the guard interval.

Forward error correction (FEC) uses error correction coding system where Concatenated Reed Solomon/Convolutional codes, and LDPC coding system are be used as FEC techniques in this paper. By spreading the coded bits over the bandwidth of the system, an efficient coding scheme could correct for the erroneous bits and thereby exploit the wideband channel's frequency diversity [9]. These channel codes improve the bit error rate performance by adding redundant bits in the transmitted bit stream that are employed by the receiver to correct errors introduced by the channel [10]. Two competitive channel codes are employed in this paper as mentioned earlier which are LDPC and concatenated RS-CC.

B. *Channel Codes under Investigation*

LDPC and concatenated codes are compared in this paper. An LDPC code is an (n, k) linear block code whose parity-check matrix H contains a relatively small proportion of 1's in comparison to 0's. An LDPC code is said to be *regular* if its parity-check matrix has the same number J of ones in every column and the same number K of ones in every row (otherwise the code is referred to as *irregular*) [13,14]. Every parity check matrix can be represented by a *Tanner graph* [16, 17]. As an example, for the parity check matrix

$$H = \begin{pmatrix} 1&1&1&0&0&0 \\ 0&1&1&1&0&0 \\ 1&0&0&0&1&1 \\ 0&0&0&1&1&1 \end{pmatrix}$$

the corresponding Tanner graph is shown in Figure 1, where the variable nodes v_i, i= 0,1,2,3,4,5 are represented by circles and the check nodes c_j, j= 0,1,2,3 are represented by squares. We employed one of the Euclidean Geometry LDPC (EG-LDPC) codes described in [18]; specifically, the class-I two-dimensional (2-D) EG-LDPC codes corresponding to $s = 5$, $m = 2$. This code is regular with block length $n = 1023$, number of information bits $k = 781$ and number of parity bits $n - k = 242$.

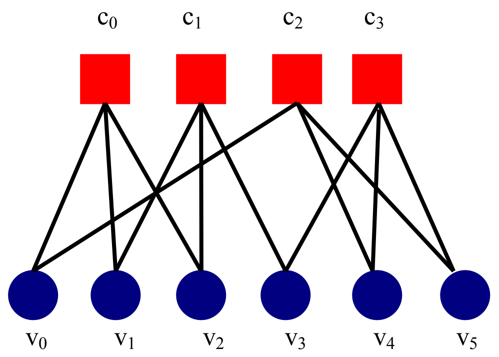

Figure 1: Tanner graph for the example parity check matrix H

Finite geometry LDPC codes have relatively good minimum distances and their Tanner graphs do not contain cycles of length 4 [18]. Finite geometry LDPC has good to very good performances compared to other structured LDPC codes while having reasonable decoding complexity. Preferably, it works very well when decoded using the iterative sum product algorithm (SPA).

In general, when LDPC coding is used in conjunction with QPSK-OFDM communication system, the formation of the log-likelihood ratios (LLRs) is an important step; we describe this below for QPSK-OFDM communication system. Assuming perfect synchronization receiver exact the useful symbol time and it will remove the cyclic prefix. After performing the frequency domain computation using FFT, and fed in to the equalizer. Assuming the delay spread of the channel is smaller than the cyclic prefix and the time variance of the channel per one OFDM symbol is negligible, the received symbol R_k in the frequency domain is given by

$$R_k = H_k A_k + W_k \tag{1}$$

where, $A_k = a_k + jb_k$ denotes the QPSK symbol, H_k is the frequency-domain gain of the k-th subchannel, and W_k is complex additive white Gaussian noise with variance σ^2 per real dimension. The LLR former is then given by

$$\lambda(a_k) = \left(\frac{2|H_k|^2}{\sigma^2}\right) Re(Y_k) \tag{2}$$

$$\lambda(b_k) = \left(\frac{2|H_k|^2}{\sigma^2}\right) Im(Y_k) \tag{3}$$

where $Y_k = R_k/H_k$ is the output of the frequency-domain equalizer.

Concatenated coding, first proposed by Forney [8], consists in cascading two or more simple codes in a serial manner, where the resulting composite code is equivalent to a much longer code in performance and lower in decoding complexity. The input bits are first coded and interleaved by an outer coder and interleaver and coded bits are then again coded and interleaved by an inner coder and interleaver [10]. Due to its simplicity and effectiveness, the technique has been adopted widely. The most common usage of the concatenated coding technique is to pair a Reed Solomon code as the outer code with a convolutional code as the inner code. Since the decoder for convolutional code is Viterbi decoder, the decoder output tends to produce errors in bursts and the Reed Solomon code is able to correct these error bursts. The convolutional codes with soft decision decoding perform better for relatively low input SNR where a hard-decision block decoder then correcting the remaining errors. RS (255,239,8) scheme and rate-½ convolutional coding were used in this paper with constraint length 3 and code generators (6, 7) in octal notation. For the Viterbi decoder, a traceback length of 42 was used.

III SIMULATED OFDM SPECIFICATION

To obtain the bit error rate performance of the OFDM communication system, the following parameters have been used. The three fading channels used in this paper are the COST 207 Typical Urban/Bad Urban channels with 12 paths, and the Winner scenario 2.8 channel model with 20 paths. The Winner model is structured for indoor and outdoor environments for the 5 GHz range of frequency [11].

The receiver's mobility was taken into consideration in this paper. 100Hz and 400Hz frequency shifts were considered and the corresponding velocities are 30 km/hr and 120 km/hr respectively.

Table I: OFDM simulation parameters

Parameters	Value
FFT size	256
Bandwidth	2.5MHz
Modulation	QPSK,BPSK
Channel Coding	Rate-½ CC and RS (255,239,8)
	Euclidean Geometry LDPC (1023,781)
Channels	COST 207 TU and BU with 12 paths and Winner 2.8 scenario with 20 paths

IV SIMULATION RESULTS

This section documents the link level BER performance of the simulated coded OFDM systems over the fading channels mentioned above.

The complete set of propagation paths between transmitter and receiver form the multipath channel in each case. Each path can be characterized by three parameters: delay, attenuation, and phase shift. It is expected to have worse performance for multipath Rayleigh fading channels. Doppler frequency is a significant factor in flat fading. Doppler spread is caused by the differences in Doppler shifts of different components of the received signal, if the transmitter or receiver is in motion. The theoretical performance of QPSK-OFDM in flat Rayleigh fading channels for various speeds, together with the associated mathematical derivations, are given in the [1].

Table II: COST 207 BU/TU channel parameters

Tap No.	Relative Delay (μs)		Fading (dB)	Fading (dB)
	BU	**TU**	**BU**	**TU**
0	0	0	-7	-4
1	0.2	0.2	-3	-3
2	0.4	0.4	-1	0
3	0.8	0.6	0	-2
4	1.6	0.8	-2	-3
5	2.2	1.2	-6	-5
6	3.2	1.4	-7	-7
7	5.0	1.8	-1	-5
8	6.0	2.4	-2	-6
9	7.2	3.0	-7	-9
10	5.0	3.2	-10	-11
11	10.0	5.0	-15	-10

The power delay profile of the COST 207 Typical Urban and Bad Urban channels are listed in Table II as given in the [15] and un-coded BER performance over these channels is given in [12].

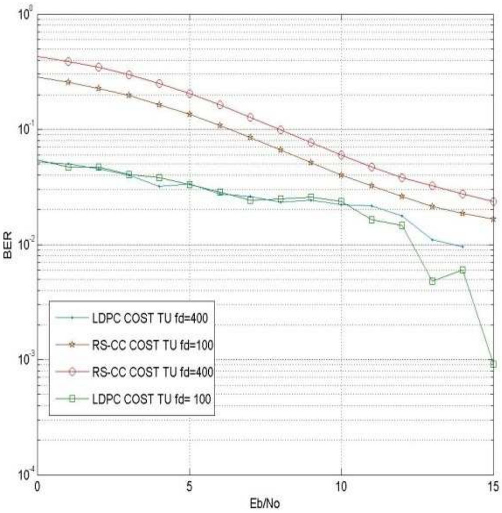

Figure 5: BER for QPSK-OFDM With RS-CC and LDPC coding systems Over COST207 TU

Figure 5 shows the performance of QPSK-OFDM using concatenated Convolutional/Reed Solomon coding system and LDPC coding system over COST 207 Typical Urban. BER performance of LDPC coded COST TU is better than the concatenated Convolutional/Reed Solomon coded QPSK-OFDM. As we can see from the Figure 5, when the Doppler frequency is increased the error floor in the BER becomes higher.

Figure 6 shows the BER performance of concatenated Convolutional/ Reed Solomon coded QPSK-OFDM and LDPC coded QPSK-OFDM over the COST 207 Bad Urban channel. As depicted in Figure 6, LDPC coded QPSK exhibits better performance over the concatenated Reed Solomon - convolutional coded QPSK-OFDM over COST 207 Bad Urban channel.

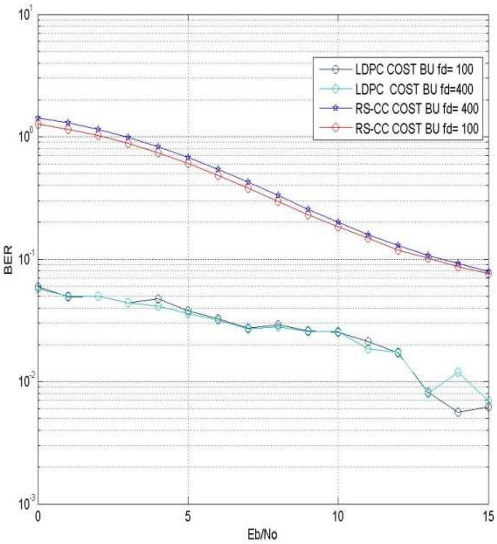

Figure 6: concatenated RS/CC coded QPSK OFDM System performance over COST 207 BU channel

Table III: Winner vehicular 2.8 channel parameters

Tap index	Relative Delay(ns)	Average Power(dB)
1	0	-1.25
2	10	0
3	40	-0.38
4	60	-0.1
5	85	-0.73
6	110	-0.63
7	135	-1.78
8	165	-4.07
9	190	-5.12
10	220	-6.34
11	245	-7.35
12	270	-8.86
13	300	-10.1
14	325	-10.5
15	350	-11.3
16	375	-12.6
17	405	-13.9
18	430	-14.1
19	460	-15.3
20	485	-16.3

Figure 7 shows the BER performance of concatenated Convolutional/Reed Solomon and LDPC coded QPSK OFDM over Winner vehicular channel. As we can see from the Figure 7 performance of LDPC coded QPSK OFDM having better performance.

The best performance was achieved over the Winner scenario 2.8 NLOS channel when using the power delay profile values given in Table III [15]. In comparison to the performances obtained over the COST 207 channels, the BER curve for the Winner vehicular NLOS model was lower according to Figure 7 and this behaviour is expected though the Winner scenario 2.8 has 20 taps. The maximum delay it incurs is 485 ns. The system performance can be further improved by spreading the modulated symbols using Hadamard-Walsh (HW) codes [19].

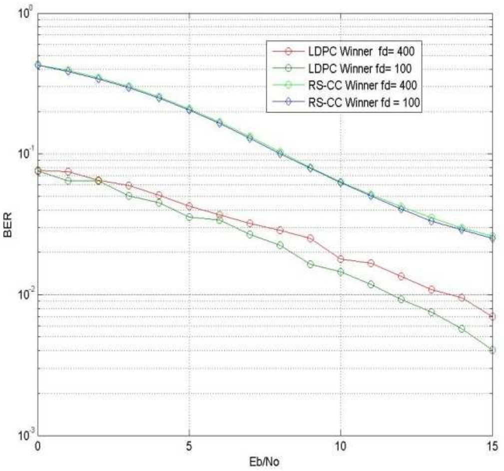

Figure 7: BER Performance of concatenated RS-CC and LDPC coded QPSK-OFDM over Winner Scenario. 2.8

The code rate of the LDPC code used in this paper is 781/1023 = 0.76. The Reed-Solomon code has code rate 239/255=0.93; hence the code rate of the concatenated RS/CC is 0.46. Since the code rate is significantly higher for the LDPC-coded system, it exhibits a significant throughput advantage in addition to improved error-correction performance.

V CONCLUSION

In this paper we presented link level BER performance of coded QPSK-OFDM operating over frequency selective multipath Rayleigh fading channels. It was shown that the LDPC-coded system has an improved BER performance compared to the concatenated convolutional/Reed-Solomon coded system. The best BER performance was obtained over the Winner channel (smallest delay spread) with LDPC coding. As expected, the worst performance was achieved over the COST BU channel.

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