Application of wireless network control to a two inverted pendulum system

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Abstract — **This paper applies a recent multi-observer approach for IEEE802.11b wireless networked control to a two inverted pendulum(TIP) system. This relies on multiple observers, corresponding to network induced delays that are multiples of the measurement sample interval. State estimates for packet-based linear quadratic regulation are selected according to the round-trip delay(RTD). This quality of service(QoS) measure for the wireless channel is characterized by an Inverse Gaussian(IG) distribution which was derived experimentally. Linear matrix inequalities(LMI) are used in the design of a stable closed-loop wireless networked control system for the TIP system. Simulation results demonstrate that the multi-observer approach can indeed compensate for wireless network induced delay and improve the performance of the wireless networked TIP system, thereby meeting the hard-wired performance specification.**

Keywords — **Two inverted pendulum, Multi-observer, Wireless networked control system, Round-trip delay, Inverse Gaussian distribution, Linear-Quadratic Regulator**

I Introduction

Wireless Networked Control Systems (WNCS) have generated considerable interest in both the industry and research communities. Although the majority of reported applications have been on process monitoring, the demand for WNCS is expected to grow because of the significant cost savings accruing. Thus, a wireless communication infrastructure is ideal in harsh environments where chemicals, vibrations, or moving parts could physically damage the cabling [1].

Much effort has been put into understanding the effects of the wireless channel and the protocol on the closed-loop control performance in a WNCS [2]. Some experimental studies have also examined the applicability of the WNCS under real operating conditions. [3] performed experiments to understand the statistical properties of packet loss and bit-error rate for the design and simulation of industrial wireless local area network protocols. Another paper [4] recommended that the optimal selection of control parameters, such as the sample period, should depend on the choice of communication parameters. This is an example of co-design [5]. Here the proper approach to WNCS design integrates both communications and control for the best closed-loop control while also optimizing the communication channel performance.

The idea of using an observer in WNCS was proposed early on by [6], however this only dealt with constant time-delays. A subsequent paper [7] introduced an LMI gridding approach in the design of both a non-stationary observer and feedback gains for systems with time-varying sampling and delay. More recently, a new multi-observer approach to WNCS was described [8], with both Lost Sample Observers (LSO) and State Prediction Observers (SPO) to address both time-varying delays and packet dropouts in WNCS. Here closed-loop stability was proven using results from [9] and the well-known cart-mounted inverted pendulum was used to verify the effectiveness of the proposed scheme for IEEE802.11b wireless control in simulation.

This paper reports on the application of this multi-observer based WNCS to a significantly more challenging system. This comprises two inverted pendulums (TIP) coupled by a spring [10]. The round-trip delay is modelled by an Inverse Gaussian distribution as a QoS measure for adjusting the sample rate according to wireless channel conditions. The design is presented along with results from Monte-Carlo simulations comparing the closed-loop control performance with the baseline hardwired LQ control.

This paper is organised as follows. Section 2 describes the nonlinear TIP application and gives the linear state model needed for hardwired LQ control design. The next section outlines the characteristics of the random round-trip delay encountered with IEEE802.11b wireless control and contains the Inverse Gaussian distribution model. Section 4 briefly presents the multi-observer WNCS approach and the closed-loop design of the wireless networked TIP system. The results of the Monte-Carlo simulation studies are presented in the next section, followed by some conclusions and suggestions for future work.

II Two Inverted Pendulum(TIP) system

The TIP laboratory system [10] consists of two identical cart-mounted inverted pendulums coupled by a spring and is an ideal platform for studying the modelling and control of nonlinear unstable systems. Fig. 1 gives a simple schematic of the TIP control system. The two inputs $u_1 = \ddot{\alpha}_1$, $u_2 = \ddot{\alpha}_2$ are the accelerations of cart 1, cart 2 respectively. The measured outputs are $\mathbf{y} = [\alpha_1\ \theta_1\ \alpha_2\ \theta_2]^T$, where α_1, α_2 are the lateral displacements of cart 1 and cart 2, with θ_1 and θ_2 being the angular deflections of the two pendulums. Here the aim is to stabilise the TIP system so that the positions of the two carts are controlled quickly, while regulating the two pendulums to maintain their vertical positions.

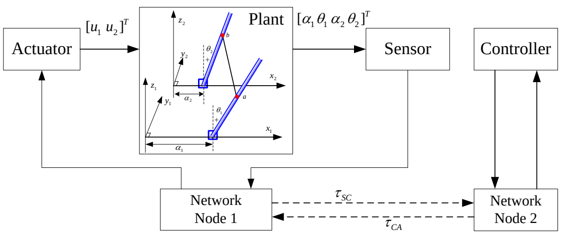

Fig. 1: Schematic of the two inverted pendulum (TIP) control system

The complete nonlinear dynamic model of the TIP system can be linearized about the unstable equilibrium point $\mathbf{x}_0 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$, to produce the continuous system model of equation (1) for the system parameter values listed in Table 1.

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}u \\ \mathbf{y} &= \mathbf{C}\mathbf{x} \end{aligned} \tag{1}$$

Here:

$$\mathbf{x} = \begin{bmatrix} \alpha_1 & \dot{\alpha}_1 & \theta_1 & \dot{\theta}_1 & \alpha_2 & \dot{\alpha}_2 & \theta_2 & \dot{\theta}_2 \end{bmatrix}^T$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -13.97 & 0 & 18.49 & 0 & 13.97 & 0 & 5.59 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 13.97 & 0 & 5.59 & 0 & -13.97 & 0 & 18.49 & 0 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 2.4547 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 2.4547 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Table 1: Parameter values of TIP system

Symbol	Description	Value
$m_i(i=1,2)$	mass of the pendulum	$0.1527kg$
l_i (i=1,2)	distance from the pivot to the centre of mass of the pendulum	$0.3215m$
h	distance from end point of spring to pivot point of pendulum	$0.4m$
d	distance between the two pendulum rods	$0.187m$
L	natural length of the spring	$0.181m$
k_s	spring constant	$21.77\ N/m$
g	gravitational constant	$9.81\ m/s^2$
$J_i(i=1,2)$	rotational inertial	$0.02\ kg \cdot m^2$

The eigenvalues of the $\mathbf{A}$ matrix are {-4.9071, -3.5922, 4.9071, 3.5922, 0, 0, 0, 0}, showing that the system is open-loop unstable, thereby forming challenging control task for the WNCS. Fortunately the open-loop system is both controllable and observable.

III Characteristics of IEEE802.11b network time-delay

In order to quantify and analyse the characteristics of the time delays in a practical WNCS, the round-trip delays(RTD) [8] are recorded for an IEEE802.11b under the controlled wireless conditions provided by a reverberation chamber. The main components of the RTD are τ_{sc}, the time-delay between sensor and controller and τ_{ca} the time-delay between controller and actuator. For statistical modelling of the measured delay data produced by multiple trials, it was found that the Inverse Gaussian (IG) probability density distribution in equation (2) gave the best fit.

$$f(x;\mu,\lambda) = \left[\frac{\lambda}{2\pi x^3}\right]^{1/2} \exp\left(\frac{-\lambda(x-\mu)^2}{2\mu^2 x}\right) \tag{2}$$

where $\mu > 0$ is the mean and $\lambda > 0$ is the shape parameter. The estimated values of these two parameters and their standard errors are given in Table 2. This delay model supports the realistic simulation studies of the IEEE802.11b based WNCS for the two inverted pendulum system given later in section 5.

Table 2: Parameter estimation and standard error for μ and λ

Parameter	Values	Standard Error
μ	8.23547×10^{-3}	8.0155×10^{-5}
λ	1.79659×10^{-3}	1.15502×10^{-5}

IV Closed-loop design of wireless networked control system

Our wireless control of the two inverted pendulum system involves an event driven controller, and time-driven sensor. Invoking the Separation Principle allows the design of the multi-observer system, for handling the wireless induced time delays, and that of the linear quadratic control to be divided into two distinct problems. In the resulting time-varying sampled system, if both the observer and controller are stable, the overall closed-loop system is also stable [7].

A simple schematic is shown in Fig. 2 to illustrate the overall TIP wireless control system. The optimum LQ gain matrix F of the state-feedback controller, required to satisfy the performance criteria given in Table 3, was calculated for the discrete TIP system model derived from equation (1) with a sampling time, T_s, of 0.01s and a wired channel (i.e. ideal wireless). The multi-observer WNCS approach can handle incomplete information on the current states as well as providing future state predictions.

In the sensor block, the outputs of the plant (TIP system) are sampled every T_s *seconds* and the current states $\hat{x}(k)$ are evaluated by the current observer module. Subsequently, the current state packet is transmitted through the IEEE802.11b wireless network.

The controller block contains a set of observers and the optimal gain LQ matrix module. Each received network packet $\hat{x}(k)$ is fed to the multi-observer module in which the future states of the plant can be predicted. Each observer corresponds to a discrete system with a different sampling period T_o. For simplicity, integral multiples of T_s were used in this paper. Thus, the predicted state estimates $\hat{x}(k+1)$, $\hat{x}(k+2)$, ... , $\hat{x}(k+N)$ were produced, and used to compute the N predicted control action candidates $u(k+1)$, $u(k+2)$, ... , $u(k+N)$, in the optimum gain matrix module. A packet containing the current and sequence of predicted state estimates is then transmitted to the actuator, so-called packet-based control.

The actuator block contains a control selection module for choosing the most suitable control action. Note that the sensor and controller are clock synchronized, and that all transmitted packets are time-stamped at the sensor block. When a packet arrives at the actuator, the actual network induced delay can be calculated conveniently by simply comparing current time with its timestamp. Depending on the actual network time-delay, one control action is chosen from the packet of candidate control actions and sent to the plant. The selection strategy used for choosing the control action is given in equation (3), where $1 \le i \le N$. The T_{delay} represents the round-trip delay (RTD) while $u(\cdot)$ is the selected control action.

$$\begin{array}{ll} IF & T_{delay} \in [(i-1)T_s \;\; iT_s] \\ THEN & u(\cdot) = u(k+i) = -F\hat{x}(k+i) \end{array} \tag{3}$$

Given a sampling period T_s, the discrete system matrices A_k, B_k, C_k can be selected from a group of candidate ones A_k^j, B_k^j, C_k^j , where $j = T_s$, $2T_s$, ..., NT_s. The overall model of the before multi-observer based WNCS system is represented by equation (4). Here $\hat{x}(k+1)$ is the one-step ahead prediction of the plant state and L_k^j are the observer gain matrices at sampling instant t_k.

$$\begin{cases} x(k+1) = A_k^j x(k) + B_k^j u(k) \\ y(k) = C_k^j x(k) \\ \hat{x}(k+1) = A_k^j \hat{x}(k) + B_k^j u(k) + L_k^j (y(k) - C_k^j \hat{x}(k)) \\ \qquad\qquad = (A_k^j - L_k^j C_k^j)\hat{x}(k) + B_k^j u(k) + L_k^j y(k) \\ u(k) = -F\hat{x}(k) \end{cases} \tag{4}$$

In order to design a stable observer, Lyapunov stability results and LMI theory were applied to achieve a feasible solution. The matrix inequalities in equation (5) can be used to guarantee stability [8].

$$\begin{cases} \begin{bmatrix} P & (PA_k^j - M_k^j C_k^j)^T \\ PA_k^j - M_k^j C_k^j & P \end{bmatrix} > 0 \\ P > 0 \end{cases} \tag{5}$$

The overall multi-observer control system is then stable if the Lyapunov matrix P is symmetrical

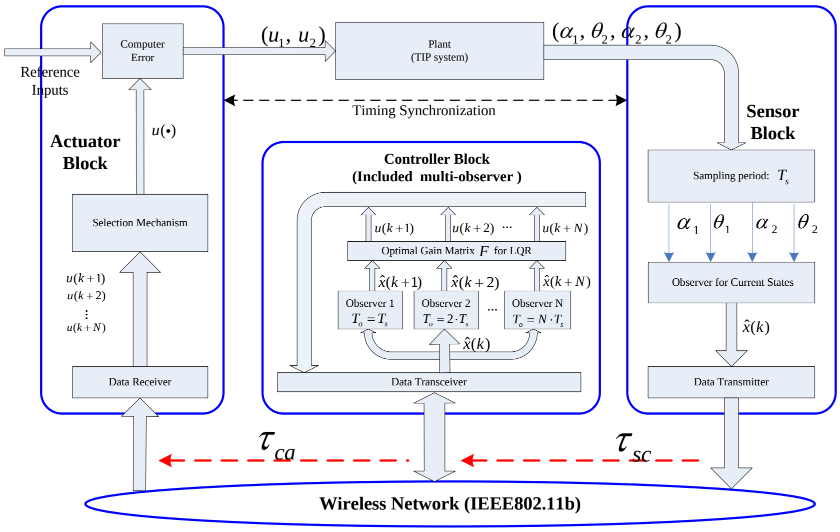

Fig. 2: A schematic of TIP wireless networked control system

and if M_k^j can be found which satisfy these matrix inequalities given by equation (5), where $M_k^j = PL_k^j \rightarrow L_k^j = P^{-1}M_k^j$. Thus, If the values of P and M_k^j are known, the L_k^j can then be evaluated subsequently.

V Numerical Simulation and Analysis

The simulations incorporated time-varying delays without giving consideration to packet losses. The random delay information generated from the IG distribution model was introduced into the feedback channel to confirm that the multi-observer approach with delay compensation could perform effectively and maintain stability.

A Monte-Carlo simulation analysis was performed to ensure that the statistical characteristics of the simulated network induced delay approximated the actual measured network delays of wireless network. To assess the performances, the criteria listed in Table 3 were used. The multi-observer can theoretically handle any size of network induced delay. Here, for simplicity the analysis was restricted to 3 observers (namely $N = 3$). Because the sampling period T_s of TIP system is 0.01s, T_o=0.01s, 0.02s and 0.03s was used to design three observers to predict one, two and three steps ahead.

Table 3: Step response performance criteria for the closed-loop WNCS

Output	Criterion	Value
α_1, α_2	Settling Time (t_s)	$< 4s$
	Overshoot (σ_p)	$< 30\%$
	steady state error (e_s)	$< 1\%$
θ_1, θ_2	Settling Time (t_s)	$< 4s$
	Overshoot (σ_p)	<0.4rad(23^o)
	steady state error (e_s)	<0.017rad(1^o)

The corresponding optimum LQ gain matrix $F_{0.01}$ and the observer gain matrices $L_{0.01}, L_{0.02}, L_{0.03}$ produced by the design procedure of section 2 were:

$$F_{0.01} = \begin{bmatrix} -25.7288 & -41.7243 & -150.0371 & -27.5321 & -65.0782 & -15.6797 & -37.7623 & -6.7933 \\ -65.0782 & -15.6797 & -37.7623 & -6.7933 & -25.7288 & -41.7243 & -150.0371 & -27.5321 \end{bmatrix}$$

$$L_{0.01} = \begin{bmatrix} 1.0057 & 0 & 0 & 0 \\ 0.1979 & -0.0024 & 0 & 0.0024 \\ 0 & 1.0071 & 0 & 0 \\ -0.1423 & 0.3897 & 0.1423 & 0.0579 \\ 0 & 0 & 1.0057 & 0 \\ 0 & 0.0024 & 0.1979 & -0.0024 \\ 0 & 0 & 0 & 1.0071 \\ 0.1423 & 0.0579 & -0.1423 & 0.3897 \end{bmatrix} L_{0.02} = \begin{bmatrix} 1.0163 & -0.0021 & 0 & 0.0021 \\ 0.3505 & -0.0161 & 0 & 0.0161 \\ -0.0050 & 1.0265 & 0.0050 & 0.0028 \\ -0.2975 & 0.7712 & 0.2975 & 0.1251 \\ 0 & 0.0021 & 1.0163 & -0.0021 \\ 0 & 0.0161 & 0.3505 & -0.0161 \\ 0.0050 & 0.0028 & -0.0050 & 1.0265 \\ 0.2975 & 0.1251 & -0.2975 & 0.7712 \end{bmatrix} L_{0.03} = \begin{bmatrix} 1.0179 & -0.0051 & 0 & 0.0051 \\ 0.4070 & -0.0386 & 0.0033 & 0.0386 \\ -0.0116 & 1.0448 & 0.0116 & 0.0067 \\ -0.4638 & 1.0997 & 0.4638 & 0.1997 \\ 0 & 0.0051 & 1.0179 & -0.0051 \\ 0.0033 & 0.0386 & 0.4070 & -0.0386 \\ 0.0116 & 0.0067 & -0.0116 & 1.0448 \\ 0.4638 & 0.1997 & -0.4638 & 1.0997 \end{bmatrix}$$

The initial states of TIP system were randomly chosen within the following limits:

$$\begin{aligned} &\alpha_1 \in [-0.1\ 0.1]\, m,\ \alpha_2 \in [-0.1\ 0.1]\, m \\ &\theta_1 \in [-0.15\ 0.15]\, rad,\ \theta_2 \in [-0.15\ 0.15]\, rad \end{aligned} \tag{6}$$

a) *Step and Impulse Responses of the Closed-loop TIP WNCS*

A step change of 0.2 m was assumed in both α_1 and α_2, while the pendulum angles θ_1 and θ_2 were both regulated to 0 rad. Fig. 3 shows the averaged closed-loop step responses from 200 Monte-Carlo simulations for the four outputs. In addition, to assess the disturbance rejection properties, an impulse was applied to simulate external interference, such as a wind gust on the two pendulums. Fig. 4 shows the averaged impulse responses from 200 Monte-Carlo simulations with the pendulum angle θ_1 disturbed at $t = 1second$. In both Fig. 3 and Fig. 4, graphs of the response for a hardwired LQ controller, designed to meet the specification in Table 3, and for the same LQ controller, in a WNCS without delay compensation, are included for comparison purpose.

From these graphs, several observations can be made:

1. The closed-loop TIP system with the multi-observer WNCS remains stable, and its performance still meets the desired design criteria in Table 3.
2. For the WNCS performances of the closed-loop LQ step and impulse responses with the multi-observers are better than those without the delay compensation.
3. The responses of the TIP system with the multi-observer WNCS are much closer to those with an ideal communication channel (hardwired case) than for the WNCS with LQ control alone.

b) Statistical Performance Analysis of TIP System

Analysing the raw data from the step responses, the statistical results on the closed-loop performances from 1000 Monte-Carlo simulations are summarised in Table 4.

Table 4: Failure probabilities from 1000 experiments.

Parameters		WNCS				Ideal Channel	
		LQR with multi-observer		LQR		LQR	
t_s	α_1	0	√	4.3%	×	0	√
	α_2	0	√	4.4%	×	0	√
	θ_1	0	√	2.0%	×	0	√
	θ_2	0	√	2.1%	×	0	√
σ_p	α_1	3.4%	×	22.1%	×	0	√
	α_2	3.0%	×	22.4%	×	0	√
	θ_1	2.8%	×	18.7%	×	0	√
	θ_2	2.7%	×	18.2%	×	0	√
e_s	α_1	0	√	0.1%	×	0	√
	α_2	0	√	0.2%	×	0	√
	θ_1	0	√	0%	√	0	√
	θ_2	0	√	0%	√	0	√

Several conclusions can be drawn:

1. By design, all the closed-loop step responses satisfy the required performance criteria with an ideal wireless communication channel.
2. The probability of failure of a wireless control system with multi-observers is smaller than the one without the multi-observer under the same network conditions.

c) Quantitative Analysis of the Control Quality

The Integrated-Time-Area-Error ($ITAE$) index was used to quantify the multi-observer WNCS performance. This is defined as equation (7).

$$ITAE = \sum_{k=k_0}^{k_f} k|e_k| \tag{7}$$

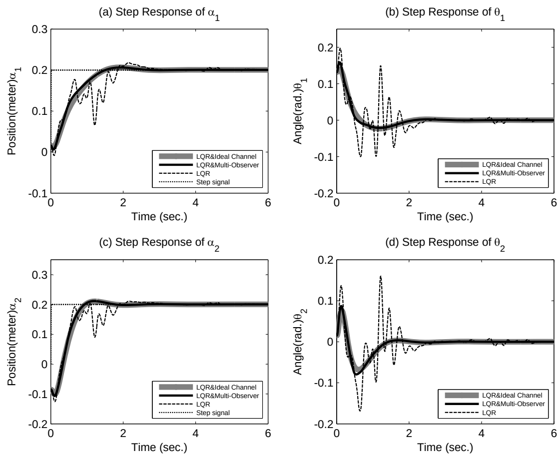

Fig. 3: Comparison of averaged step responses from 200 Monte-Carlo simulations for the multi-observer WNCS, the hardwired LQ control system and a WNCS without multi-observer delay compensation (initial states: α_1=0.0154m, θ_1=0.1319rad, α_2=-0.0850m, θ_2=0.0152rad).

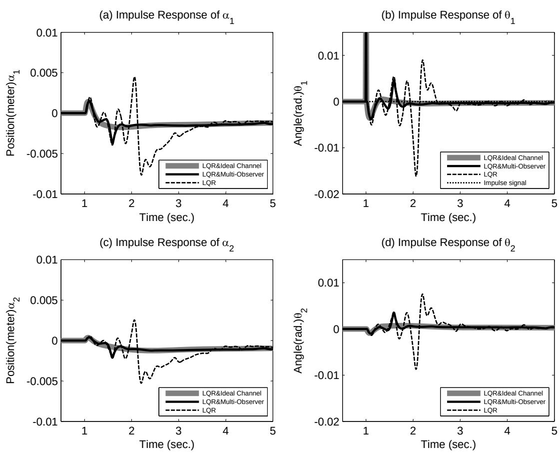

Fig. 4: Comparison of averaged impulse responses from 200 Monte-Carlo simulations for the multi-observer WNCS, the hardwired LQ control system and a WNCS without multi-observer delay compensation.

Here k_0 and k_f are the initial and final sample indices and e_k is the closed-loop step response error. Two other indices, $ITAE_{ob}$ and $ITAE_{nob}$, were used to represent the WNCS performances compared to the ideal channel case.

$$ITAE_{ob} = \log_{10}(\frac{ITAE_M}{ITAE_I}) \tag{8}$$

$$ITAE_{nob} = \log_{10}(\frac{ITAE_W}{ITAE_I}) \tag{9}$$

Here $ITAE_I$ represents the closed-loop control performance with an ideal channel, $ITAE_M$ represents the closed-loop WNCS performance with the multi-observer and $ITAE_W$ measures the closed-loop WNCS performance without any delay compensation. Smaller $ITAE_{ob}$ and $ITAE_{nob}$ values indicate that the WNCS step-response is close to the ideal case (no time-delay).

For 1000 Monte-Carlo simulation trials, the average values found for $ITAE_{ob}$ and $ITAE_{nob}$ for α_1, θ_1, α_2, θ_2 are given in Table 5. Some useful conclusions can be drawn:

1. The values of $ITAE_{ob}$ are very small, and this implies that the performances of the TIP WNCS with the multi-observer are very close to those with the ideal channel (hardwired case).

2. The $ITAE_{ob}$ values are smaller than the $ITAE_{nob}$ ones. This means that the performance of WNCS with the multi-observer delay compensation is better than the one with the LQ control alone.

Table 5: Average $ITAE_{ob}$ and $ITAE_{nob}$ values from 1000 trials

	α_1	θ_1	α_2	θ_2
$ITAE_{ob}$	0.0265	0.0373	0.0276	0.0389
$ITAE_{nob}$	0.2174	0.3036	0.2179	0.3036

VI Conclusions and future work

In this paper, WNCS for a two inverted pendulum system was built to evaluate the multi-observer approach for compensating network induced random delay described in [8]. Without loss of generality, the initial states of plant were randomly selected, and the network time-delays from an IG distribution model were used in the simulations. Extensive numerical simulations were performed on the step and impulse responses of the closed-loop TIP system. These results showed that the multi-observer WNCS approach could greatly improve the performance of the TIP WNCS, providing further evidence that the multi-observer approach is a valuable method to compensate for wireless network induced delay.

The architecture of the TIP WNCS shown in Fig. 2, can be further improved. If the four sensors are not co-located, the sampling data will have to be sent via different packets. This will significantly affect the characteristics of wireless network, introduce uncertainty into the TIP WNCS, and degrade its performance. Packet losses will also be considered in future work.

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